Solution

$\int_0^2 2\pi x^3\sqrt{1+\left(3x^2\right)^2}\,dx=\frac{\pi\left(145\sqrt{145}-1\right)}{27}$ (Decimal: $203.04360\ldots$)

Steps

$\int_0^2 2\pi x^3\sqrt{1+\left(3x^2\right)^2}\,dx$

Take the constant out: $\int a\cdot f(x)dx=a\cdot\int f(x)dx$

$=2\pi\cdot\int_0^2 x^3\sqrt{1+\left(3x^2\right)^2}\,dx$

Apply u − substitution: $u=1+\left(3x^2\right)^2$ *Show Steps*

$=2\pi\cdot\int_1^{145}\frac{\sqrt{u}}{36}\,du$

Take the constant out: $\int a\cdot f(x)dx=a\cdot\int f(x)dx$

$=2\pi\frac{1}{36}\cdot\int_1^{145}\sqrt{u}\,du$

Apply radical rule: $\sqrt{a}=a^{\frac{1}{2}}$

$=2\pi\frac{1}{36}\cdot\int_1^{145}u^{\frac{1}{2}}du$

Apply the Power Rule: $\int x^a dx=\frac{x^{a+1}}{a+1},\quad a\neq -1$

$=2\pi\frac{1}{36}\left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^{145}$

Simplify $2\pi\frac{1}{36}\left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^{145}$: $\frac{\pi}{18}\left[\frac{2u^{\frac{3}{2}}}{3}\right]_1^{145}$ *Show Steps*

$=\frac{\pi}{18}\left[\frac{2u^{\frac{3}{2}}}{3}\right]_1^{145}$

Compute the boundaries: $\frac{290\sqrt{145}}{3}-\frac{2}{3}$ *Show Steps*

$=\frac{\pi}{18}\left(\frac{290\sqrt{145}}{3}-\frac{2}{3}\right)$

Simplify

$\pi\left(145\sqrt{145}-1\right)$

$$= \frac{\pi(145\sqrt{145} - 1)}{27}$$