Symbolab

Solution

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+1}dx=6\left(\frac{1}{7}x^{\frac{7}{6}}-\frac{1}{5}x^{\frac{5}{6}}+\frac{1}{3}\sqrt{x}-\sqrt[6]{x}+\arctan\left(\sqrt[6]{x}\right)\right)+C$$

Steps

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+1}dx$$

Apply u − substitution: $u=\sqrt[6]{x}$ *Show Steps*

$$=\int \frac{6u^8}{u^2+1}du$$

Take the constant out: $\int a\cdot f(x)dx=a\cdot\int f(x)dx$

$$=6\cdot\int \frac{u^8}{u^2+1}du$$

Long division $\frac{u^8}{u^2+1}$: $u^6-u^4+u^2-1+\frac{1}{u^2+1}$ *Show Steps*

$$=6\cdot\int u^6-u^4+u^2-1+\frac{1}{u^2+1}du$$

Apply the Sum Rule: $\int f(x)\pm g(x)dx=\int f(x)dx\pm\int g(x)dx$

$$=6\left(\int u^6du-\int u^4du+\int u^2du-\int 1du+\int\frac{1}{u^2+1}du\right)$$

$\int u^6du=\frac{u^7}{7}$ *Show Steps*

$\int u^4du=\frac{u^5}{5}$ *Show Steps*

$\int u^2du=\frac{u^3}{3}$ *Show Steps*

$\int 1du=u$ *Show Steps*

$\int \frac{1}{u^2+1}du=\arctan(u)$ *Show Steps*

$$=6\left(\frac{u^7}{7}-\frac{u^5}{5}+\frac{u^3}{3}-u+\arctan(u)\right)$$

Substitute back $u=\sqrt[6]{x}$

$$=6\left(\frac{\left(\sqrt[6]{x}\right)^7}{7}-\frac{\left(\sqrt[6]{x}\right)^5}{5}+\frac{\left(\sqrt[6]{x}\right)^3}{3}-\sqrt[6]{x}+\arctan\left(\sqrt[6]{x}\right)\right)$$

Simplify $6\left(\frac{\left(\sqrt[6]{x}\right)^7}{7}-\frac{\left(\sqrt[6]{x}\right)^5}{5}+\frac{\left(\sqrt[6]{x}\right)^3}{3}-\sqrt[6]{x}+\arctan\left(\sqrt[6]{x}\right)\right)$: $6\left(\frac{1}{7}x^{\frac{7}{6}}-\frac{1}{5}x^{\frac{5}{6}}+\frac{1}{3}\sqrt{x}-\sqrt[6]{x}+\arctan\left(\sqrt[6]{x}\right)\right)$ Show Steps

$$=6\left(\frac{1}{7}x^{\frac{7}{6}}-\frac{1}{5}x^{\frac{5}{6}}+\frac{1}{3}\sqrt{x}-\sqrt[6]{x}+\arctan\left(\sqrt[6]{x}\right)\right)$$

Add a constant to the solution

$$=6\left(\frac{1}{7}x^{\frac{7}{6}}-\frac{1}{5}x^{\frac{5}{6}}+\frac{1}{3}\sqrt{x}-\sqrt[6]{x}+\arctan\left(\sqrt[6]{x}\right)\right)+C$$