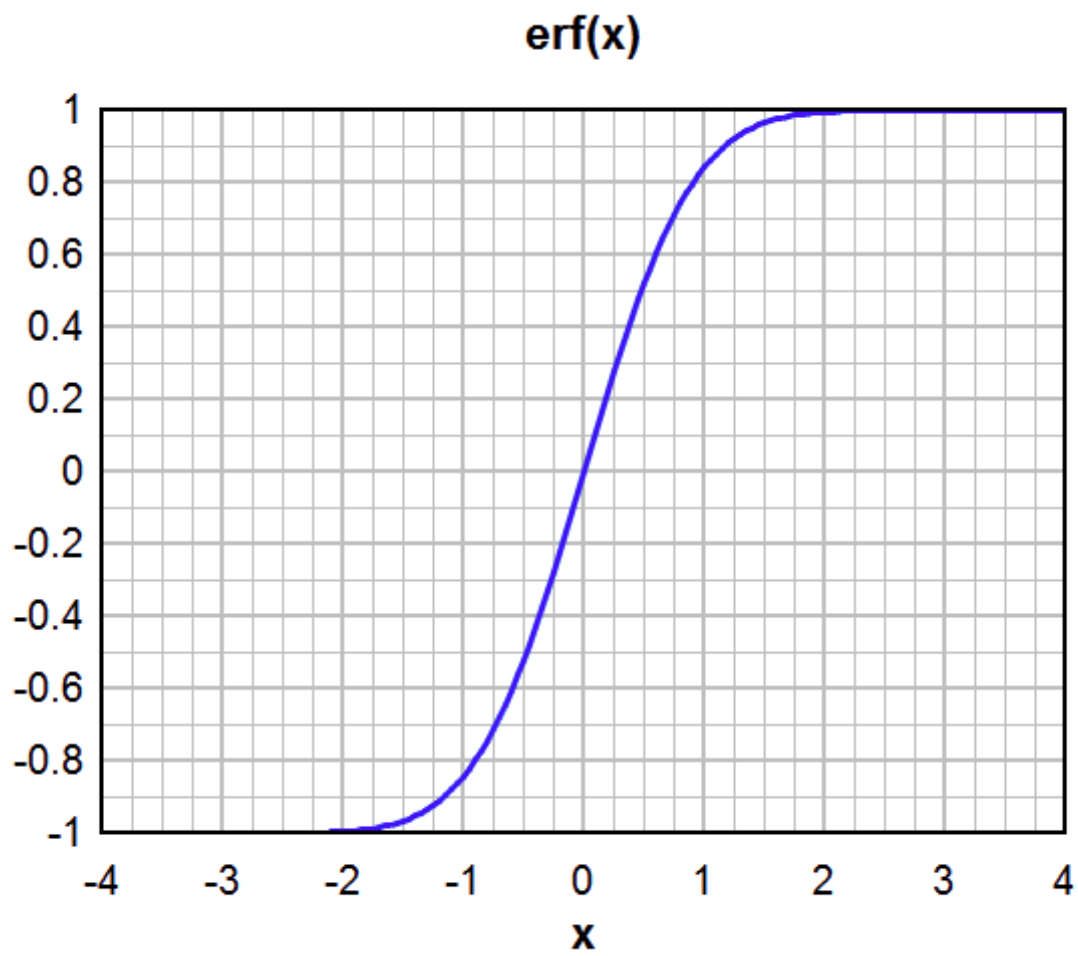


## Error Function

$$\begin{aligned}\operatorname{erf}(x) &= \frac{1}{\sqrt{\pi}} \int_{-x}^x e^{-t^2} dt \\ &= \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.\end{aligned}$$



**Tabulated values of the error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$

$x$	$\operatorname{erf} x$	$x$	$\operatorname{erf} x$	$x$	$\operatorname{erf} x$	$x$	$\operatorname{erf} x$
0.00	0.000000	0.40	0.428392	0.80	0.742101	1.20	0.910314
0.01	0.011283	0.41	0.437969	0.81	0.748003	1.21	0.912956
0.02	0.022565	0.42	0.447468	0.82	0.753811	1.22	0.915534
0.03	0.033841	0.43	0.456887	0.83	0.759524	1.23	0.918050
0.04	0.045111	0.44	0.466225	0.84	0.765143	1.24	0.920505
0.05	0.056372	0.45	0.475482	0.85	0.770668	1.25	0.922900
0.06	0.067622	0.46	0.484655	0.86	0.776100	1.26	0.925236
0.07	0.078858	0.47	0.493745	0.87	0.781440	1.27	0.927514
0.08	0.090078	0.48	0.502750	0.88	0.786687	1.28	0.929734
0.09	0.101282	0.49	0.511668	0.89	0.791843	1.29	0.931899
0.10	0.112463	0.50	0.520500	0.90	0.796908	1.30	0.934008
0.11	0.123623	0.51	0.529244	0.91	0.801883	1.31	0.936063
0.12	0.134758	0.52	0.537899	0.92	0.806768	1.32	0.938065
0.13	0.145867	0.53	0.546464	0.93	0.811564	1.33	0.940015
0.14	0.156947	0.54	0.554939	0.94	0.816271	1.34	0.941914
0.15	0.167996	0.55	0.563323	0.95	0.820891	1.35	0.943762
0.16	0.179012	0.56	0.571616	0.96	0.825424	1.36	0.945561
0.17	0.189992	0.57	0.579816	0.97	0.829870	1.37	0.947312
0.18	0.200936	0.58	0.587923	0.98	0.834232	1.38	0.949016
0.19	0.211840	0.59	0.595936	0.99	0.838508	1.39	0.950673
0.20	0.222703	0.60	0.603856	1.00	0.842701	1.40	0.952285
0.21	0.233522	0.61	0.611681	1.01	0.846810	1.41	0.953852
0.22	0.244296	0.62	0.619411	1.02	0.850838	1.42	0.955376
0.23	0.255023	0.63	0.627046	1.03	0.854784	1.43	0.956857
0.24	0.265700	0.64	0.634586	1.04	0.858650	1.44	0.958297
0.25	0.276326	0.65	0.642029	1.05	0.862436	1.45	0.959695
0.26	0.286900	0.66	0.649377	1.06	0.866144	1.46	0.961054
0.27	0.297418	0.67	0.656628	1.07	0.869773	1.47	0.962373
0.28	0.307880	0.68	0.663782	1.08	0.873326	1.48	0.963654
0.29	0.318283	0.69	0.670840	1.09	0.876803	1.49	0.964898
0.30	0.328627	0.70	0.677801	1.10	0.880205	1.50	0.966105
0.31	0.338908	0.71	0.684666	1.11	0.883533	1.51	0.967277
0.32	0.349126	0.72	0.691433	1.12	0.886788	1.52	0.968413
0.33	0.359279	0.73	0.698104	1.13	0.889971	1.53	0.969516
0.34	0.369365	0.74	0.704678	1.14	0.893082	1.54	0.970586
0.35	0.379382	0.75	0.711156	1.15	0.896124	1.55	0.971623
0.36	0.389330	0.76	0.717537	1.16	0.899096	1.56	0.972628
0.37	0.399206	0.77	0.723822	1.17	0.902000	1.57	0.973603
0.38	0.409009	0.78	0.730010	1.18	0.904837	1.58	0.974547
0.39	0.418739	0.79	0.736103	1.19	0.907608	1.59	0.975462

$x$	$\operatorname{erf} x$	$x$	$\operatorname{erf} x$	$x$	$\operatorname{erf} x$	$x$	$\operatorname{erf} x$
1.60	0.976348	2.10	0.997021	2.60	0.999764	3.10	0.99998835
1.61	0.977207	2.11	0.997155	2.61	0.999777	3.11	0.99998908
1.62	0.978038	2.12	0.997284	2.62	0.999789	3.12	0.99998977
1.63	0.978843	2.13	0.997407	2.63	0.999800	3.13	0.99999042
1.64	0.979622	2.14	0.997525	2.64	0.999811	3.14	0.99999103
1.65	0.980376	2.15	0.997639	2.65	0.999822	3.15	0.99999160
1.66	0.981105	2.16	0.997747	2.66	0.999831	3.16	0.99999214
1.67	0.981810	2.17	0.997851	2.67	0.999841	3.17	0.99999264
1.68	0.982493	2.18	0.997951	2.68	0.999849	3.18	0.99999311
1.69	0.983153	2.19	0.998046	2.69	0.999858	3.19	0.99999356
1.70	0.983790	2.20	0.998137	2.70	0.999866	3.20	0.99999397
1.71	0.984407	2.21	0.998224	2.71	0.999873	3.21	0.99999436
1.72	0.985003	2.22	0.998308	2.72	0.999880	3.22	0.99999473
1.73	0.985578	2.23	0.998388	2.73	0.999887	3.23	0.99999507
1.74	0.986135	2.24	0.998464	2.74	0.999893	3.24	0.99999540
1.75	0.986672	2.25	0.998537	2.75	0.999899	3.25	0.99999570
1.76	0.987190	2.26	0.998607	2.76	0.999905	3.26	0.99999598
1.77	0.987691	2.27	0.998674	2.77	0.999910	3.27	0.99999624
1.78	0.988174	2.28	0.998738	2.78	0.999916	3.28	0.99999649
1.79	0.988641	2.29	0.998799	2.79	0.999920	3.29	0.99999672
1.80	0.989091	2.30	0.998857	2.80	0.999925	3.30	0.99999694
1.81	0.989525	2.31	0.998912	2.81	0.999929	3.31	0.99999715
1.82	0.989943	2.32	0.998966	2.82	0.999933	3.32	0.99999734
1.83	0.990347	2.33	0.999016	2.83	0.999937	3.33	0.99999751
1.84	0.990736	2.34	0.999065	2.84	0.999941	3.34	0.99999768
1.85	0.991111	2.35	0.999111	2.85	0.999944	3.35	0.999997838
1.86	0.991472	2.36	0.999155	2.86	0.999948	3.36	0.999997983
1.87	0.991821	2.37	0.999197	2.87	0.999951	3.37	0.999998120
1.88	0.992156	2.38	0.999237	2.88	0.999954	3.38	0.999998247
1.89	0.992479	2.39	0.999275	2.89	0.999956	3.39	0.999998367
1.90	0.992790	2.40	0.999311	2.90	0.999959	3.40	0.999998478
1.91	0.993090	2.41	0.999346	2.91	0.999961	3.41	0.999998582
1.92	0.993378	2.42	0.999379	2.92	0.999964	3.42	0.999998679
1.93	0.993656	2.43	0.999411	2.93	0.999966	3.43	0.999998770
1.94	0.993923	2.44	0.999441	2.94	0.999968	3.44	0.999998855
1.95	0.994179	2.45	0.999469	2.95	0.999970	3.45	0.999998934
1.96	0.994426	2.46	0.999497	2.96	0.999972	3.46	0.999999008
1.97	0.994664	2.47	0.999523	2.97	0.999973	3.47	0.999999077
1.98	0.994892	2.48	0.999547	2.98	0.999975	3.48	0.999999141
1.99	0.995111	2.49	0.999571	2.99	0.999977	3.49	0.999999201
2.00	0.995322	2.50	0.999593	3.00	0.99997791	3.50	0.999999257
2.01	0.995525	2.51	0.999614	3.01	0.99997926	3.51	0.999999309
2.02	0.995719	2.52	0.999635	3.02	0.99998053	3.52	0.999999358
2.03	0.995906	2.53	0.999654	3.03	0.99998173	3.53	0.999999403
2.04	0.996086	2.54	0.999672	3.04	0.99998286	3.54	0.999999445
2.05	0.996258	2.55	0.999689	3.05	0.99998392	3.55	0.999999485
2.06	0.996423	2.56	0.999706	3.06	0.99998492	3.56	0.999999521
2.07	0.996582	2.57	0.999722	3.07	0.99998586	3.57	0.999999555
2.08	0.996734	2.58	0.999736	3.08	0.99998674	3.58	0.999999587
2.09	0.996880	2.59	0.999751	3.09	0.99998757	3.59	0.999999617

$x$	$\text{erf } x$	$x$	$\text{erf } x$	$x$	$\text{erf } x$	$x$	$\text{erf } x$
3.60	0.999999644	3.70	0.999999833	3.80	0.999999923	3.90	0.999999965
3.61	0.999999670	3.71	0.999999845	3.81	0.999999929	3.91	0.999999968
3.62	0.999999694	3.72	0.999999857	3.82	0.999999934	3.92	0.999999970
3.63	0.999999716	3.73	0.999999867	3.83	0.999999939	3.93	0.999999973
3.64	0.999999736	3.74	0.999999877	3.84	0.999999944	3.94	0.999999975
3.65	0.999999756	3.75	0.999999886	3.85	0.999999948	3.95	0.999999977
3.66	0.999999773	3.76	0.999999895	3.86	0.999999952	3.96	0.999999979
3.67	0.999999790	3.77	0.999999903	3.87	0.999999956	3.97	0.999999980
3.68	0.999999805	3.78	0.999999910	3.88	0.999999959	3.98	0.999999982
3.69	0.999999820	3.79	0.999999917	3.89	0.999999962	3.99	0.999999983

For  $x$  equal to, or greater than, 4 the following approximation may normally be used:

$$\text{erf}(x) \approx 1 - \frac{e^{-x^2}}{\sqrt{\pi}x}$$

Some *complementary* error function values for large  $x$  are:

$x$	$\text{erfc } x$
4.0	$1.59 \times 10^{-8}$
4.1	$6.89 \times 10^{-9}$
4.2	$2.93 \times 10^{-9}$
4.3	$1.22 \times 10^{-9}$
4.4	$5.01 \times 10^{-10}$
4.5	$2.01 \times 10^{-10}$
4.6	$7.92 \times 10^{-11}$
4.7	$3.06 \times 10^{-11}$
4.8	$1.16 \times 10^{-11}$
4.9	$4.30 \times 10^{-12}$

Please recall that before calculating  $\text{erf}(z)$  you may need to use this conversion formulae relating the voltage  $x$  and mean value  $m$  to the standard deviation of the noise  $\sigma$ :

$$z = \frac{x - m}{\sqrt{2}\sigma}$$

And the CD or error probability is then given by:

$$P(x) = P_e = \frac{1}{2} [ 1 + / - \text{erf}(z) ]$$

The  $+/-$  sign is used in front of the erf function depending on whether the CD is to be calculated for values less/greater than  $x$ .