APPENDIX D

MANN-KENDALL ANALYSIS

HydroGeoLogic, Inc. **Memorandum**

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SUBJECT: Mann-Kendall Analysis for the Fort Ord Site

MANN-KENDALL ANALYSIS FOR THE FORT ORD SITE

1.0 INTRODUCTION

The Mann-Kendall test was performed to evaluate the trend of TCE concentrations at various monitoring wells located at the Fort Ord site. The analysis was limited to those wells that were sampled at least on 10 different occasions and located within the area of interest, OU-1. This unit is located in the Northwest corner of the site. TCE sampling data that was collected between 1999 and 2004 was used for the analysis.

The analysis procedure is described with the aid of an example in Section 2.0. Input data and modeling assumptions are described in Section 3.0. The results are presented with a brief discussion in Section 4.0.

2.0 MANN-KENDALL ANALYSIS

2. 1 CALCULATION OF THE MANN-KENDALL STATISTIC (S)

The Mann-Kendall test is a non-parametric test for identifying trends in time series data. The test compares the relative magnitudes of sample data rather than the data values themselves (Gilbert, 1987). One benefit of this test is that the data need not conform to any particular distribution. Moreover, data reported as non-detects can be included by assigning them a common value that is smaller than the smallest measured value in the data set. The procedure that will be described in the subsequent paragraphs assumes that there exists only one data value per time period. When multiple data points exist for a single time period, the median value is used.

The data values are evaluated as an ordered time series. Each data value is compared to all subsequent data values. The initial value of the Mann-Kendall statistic, S, is assumed to be 0 (*e.g.*, no trend). If a data value from a later time period is higher than a data value from an earlier time period, S is incremented by 1. On the other hand, if the data value from a later time period is lower than a data value sampled earlier, S is decremented by 1. The net result of all such increments and decrements yields the final value of S.

Let x_1 , x_2 , … x_n represent n data points where x_j represents the data point at time j. Then the Mann-Kendall statistic (S) is given by

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} sign(x_j - x_k)$$

where :

$$\begin{aligned} sign(x_j - x_k) &= 1 \text{ if } x_j - x_k > 0 \\ &= 0 \text{ if } x_j - x_k = 0 \\ &= \text{-1 if } x_j - x_k < 0 \end{aligned}$$

A very high positive value of S is an indicator of an increasing trend, and a very low negative value indicates a decreasing trend. However, it is necessary to compute the probability associated with S and the sample size, n, to statistically quantify the significance of the trend. The procedure to compute this probability will be described in Section 2.3.

2.2 EXAMPLE

Consider the sampling program at a fictitious monitoring well where TCE concentrations of 1.5, 2.0, 1.75, and 3.0 µg/L were observed in the years 1990, 1991, 1992, and 1993 respectively. To calculate S, start with the last sample (1993). Since the TCE concentration recorded in 1993 is higher than the other three samples, increment the value of S by 1 thrice making it equal to 3. Then compare the 1993 sample with the earlier samples (1990 and 1991). The 1992 value of 1.75 µg/L is lower than the 1991 value of 2.0 µg/L, which means that we have to decrement the value of S by 1 bringing it down to 2. At the same time, the 1992 value of 1.75 µg/L is higher than the 1990 value of 1.5 µg/L, which means that S has to be incremented by 1 making it 3. Comparing the 1991 sample (2.0 µg/L) with the only earlier sample (1.5 µg/L in 1991), we realize that we have to increment S by 1, giving us a final value of 4.

2.3 CALCULATION OF PROBABILITY ASSOCIATED WITH THE MANN-KENDALL STATISTIC

Kendall (1975, p55) describes a normal-approximation test that could be used for datasets with ***more than 10 values,*** provided there are not many tied values within the data set. The test procedure is as follows:

- Calculate S as described in Section 2.1.
- Calculate the variance of S, VAR(S), by the following equation:

$$VAR(S) = \frac{1}{18}\left[n(n-1)(2n+5) - \sum_{p=1}^{g} t_p (t_p - 1)(2t_p + 5) \right]$$

where n is the number of data points, g is the number of tied groups (a tied group is a set of sample data having the same value), and t_p is the number of data points in the pth

group. In the sequence {2, 3, non-detect, 3, non-detect, 3}, we have n=6, g =2, t_1 =2 for the non-detects, and t_2 =3 for the tied value 3.

- Compute a normalized test statistic Z as follows:

$$
\begin{aligned}
Z &= \frac{S-1}{[VAR(S)]^{1/2}} \quad \text{if S } > 0 \\
&= 0 \qquad\qquad\quad \text{if S } = 0 \\
&= \frac{S+1}{[VAR(S)]^{1/2}} \quad \text{if S } < 0
\end{aligned}
$$

- Compute the probability associated with this normalized test statistic. The probability density function for a normal distribution with a mean of 0 and a standard deviation of 1 is given by the following equation:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

  Microsoft Excel function, ***NORMSDIST***(), was used to calculate this probability.
- Decide on a probability level of significance (***95% typically***).
- The trend is said to be ***decreasing*** if Z is negative and the computed probability is greater than the level of significance. The trend is said to be ***increasing*** if the Z is positive and the computed probability is greater than the level of significance. If the computed probability is less than the level of significance, there is ***no trend***.

This method, though approximate, is applicable to the OU-1 datasets because the data satisfies the various limitations, i.e., sample sizes greater than 10, and very few ties.

3.0 DATA AND ASSUMPTIONS

Monitoring well data collected between 1999 and 2004 at Fort Ord was used to perform the analysis. Samples were collected every two to three months from most wells between April 1999 and December 2004. Monitoring well MW-OU1-37-A had the least number of samples (11) while well MW-OU1-29-A had the maximum number of samples (27). On average, there were about 20 samples per monitoring well. The non-detect values were replaced with an arbitrary value of 0.01 that was smaller than the smallest measured concentration of TCE (0.061). Two key assumptions were also made in processing the data:

- When multiple samples were collected on a single day, the median value of all those samples was assumed to be the representative sample. This was a necessary step as the Mann-Kendall analysis required only one data point at a given instant of time.
- It was assumed that the variation in the sampling depths was not large enough to produce a bias in the trend statistics.

4.0 RESULTS

A spreadsheet-based macro was developed using Microsoft Visual Basic for Applications (VBA) within an Excel spreadsheet. This tool was used to process the input time-series data, perform the trend analysis, and report the results.

The TCE concentration data at 27 wells was analyzed using the aforementioned VBA macro tool. Statistically significant increasing trends were observed at four wells. Six wells did not have a trend that was statistically significant. The remaining 17 wells had a statistically significant decreasing trend. The results are shown in Table 1. The same results are presented visually in Figure 1.

The TCE plume originated from the area shaded in grey near MW-OU1-37-A and MW-OU1-12-A (Figure 1). Field studies have indicated that the plume migrated from the source in a northwesterly direction. As the plume migrates from the source, concentrations would decrease near the source with time. This is confirmed by the splash of green near the source in Figure 1, indicating a decreasing trend. As the plume diffuses away from the source, the outer wells would be expected to report increasing concentrations until the plume crosses them. The results do indicate an increasing trend in outer MW-OU1-09-A, MW-OU1-33-A, MW-OU1-32-A, and MW-OU1-25-A. Even though the trend at MW-OU1-08-A, MW-OU1-19-A, MW-OU1-21-A and MW-OU1-27-A was not significant enough, a positive Mann-Kendall statistic suggests that the TCE concentration at these wells is increasing with time.

5.0 REFERENCES

Gilbert, R.O., 1987. *Statistical methods for environmental pollution monitoring.* Van Nostrand Reinhold, New York.

Kendall, M.G., 1975. *Rank correlation methods,* 4^{th} ed. Charles Griffin, London.

Table 1 – Mann-Kendall Trend Results

Well ID	Number of Data Points	Number of Non-detects	Minimum Value	Maximum Value	Mann Kendall Statistic (S)	Normalized Test Statistic (Z)	Probability	Trend (At 95% level of significance)
MW-OU1-03-A	24	1	0.01	3.32	-252	-6.2845	1.0000	Decreasing
MW-OU1-04-A	18	0	8.3	45.75	-91	-3.4898	0.9998	Decreasing
MW-OU1-05-A	21	0	3.6	19.9	-194	-5.8938	1.0000	Decreasing
MW-OU1-06-A	21	1	0.01	6.06	-148	-4.5145	1.0000	Decreasing
MW-OU1-10-A	24	4	0.01	7.4	-95	-2.3883	0.9915	Decreasing
MW-OU1-12-A	13	0	0.31	1.51	-68	-4.2096	1.0000	Decreasing
MW-OU1-20-A	22	0	3.2	34.8	-98	-2.7927	0.9974	Decreasing
MW-OU1-22-A	24	0	2	4.64	-148	-3.7004	0.9999	Decreasing
MW-OU1-23-A	22	0	8.2	27.1	-142	-4.0339	1.0000	Decreasing
MW-OU1-26-A	22	0	20.5	54.1	-121	-3.4429	0.9997	Decreasing
MW-OU1-29-A	27	1	0.01	43.5	-211	-4.4195	1.0000	Decreasing
MW-OU1-36-A	16	0	0.48	4.64	-86	-3.9356	1.0000	Decreasing
MW-OU1-37-A	11	1	0.01	1.65	-25	-2.0241	0.9785	Decreasing
MW-OU1-38-A	13	6	0.01	0.588	-53	-3.4833	0.9998	Decreasing
MW-OU1-39-A	19	0	0.44	35.5	-151	-5.3178	1.0000	Decreasing
MW-OU1-43-A	18	1	0.01	36.3	-119	-4.5519	1.0000	Decreasing
PZ-OU1-35-A	24	1	0.01	86.4	-205	-5.1113	1.0000	Decreasing
MW-OU1-09-A	23	13	0.01	4.4	150	4.4259	1.0000	Increasing
MW-OU1-25-A	24	0	0.398	4.2	96	2.4110	0.9920	Increasing
MW-OU1-32-A	23	1	0.01	1.8	178	4.7435	1.0000	Increasing
MW-OU1-33-A	25	1	0.01	2.5	109	2.5697	0.9949	Increasing
MW-OU1-01-A	15	0	0.74	1.2	-3	-0.1979	0.5785	No Trend
MW-OU1-08-A	24	1	0.01	5.9	55	1.3895	0.9177	No Trend
MW-OU1-19-A	20	0	2.835	8.7	25	0.8440	0.8007	No Trend
MW-OU1-21-A	18	2	0.01	0.335	23	0.9117	0.8190	No Trend
MW-OU1-27-A	24	0	0.63	105	49	1.2406	0.8926	No Trend
MW-OU1-40-A	19	0	0.38	5	-45	-1.6113	0.9464	No Trend

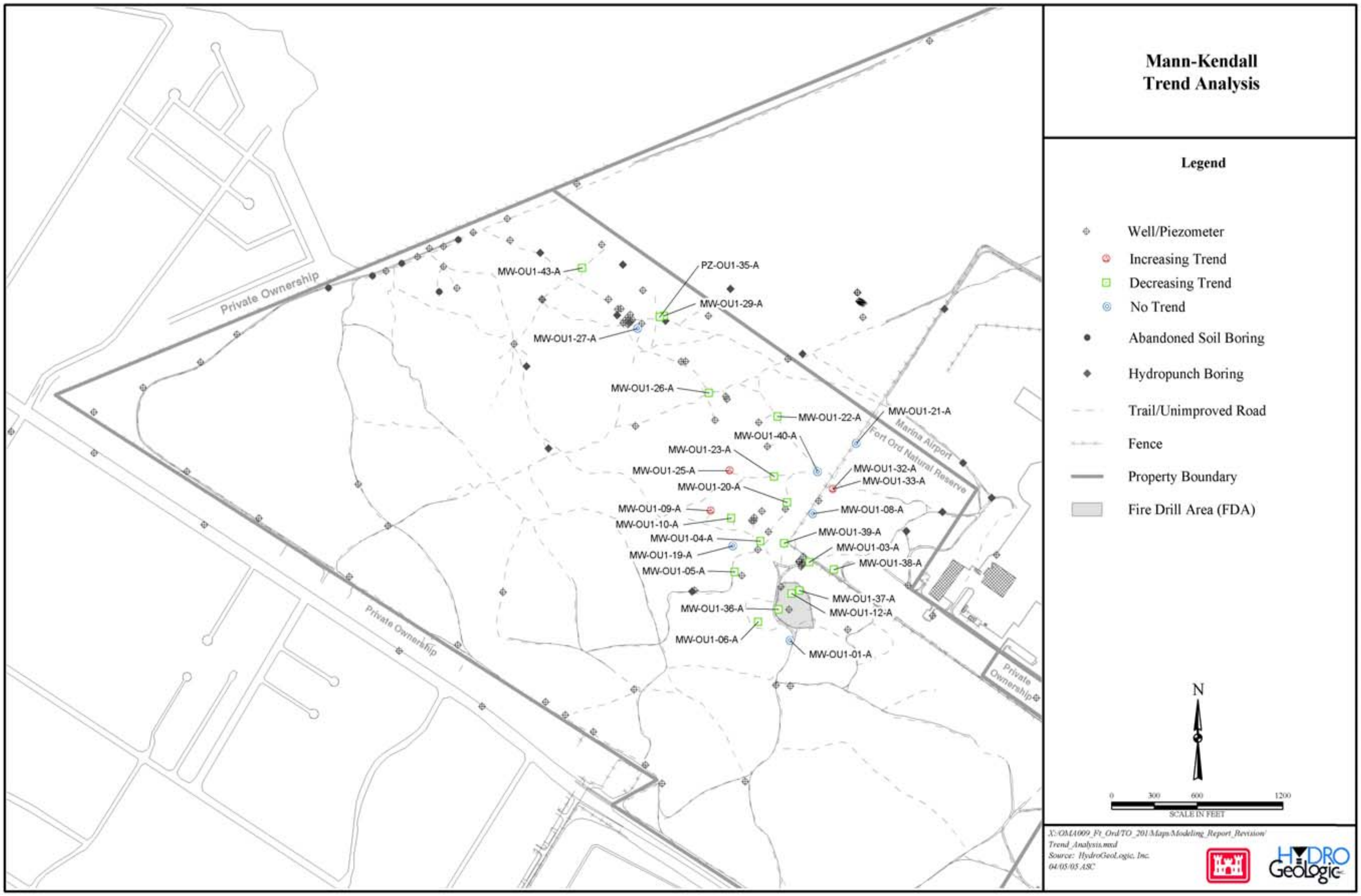

Figure 1 – Results of Mann-Kendall Trend Analysis