

**TABLE B.5**  
Critical Values for the Phillips-Perron  $Z_\rho$  Test and for the Dickey-Fuller Test  
Based on Estimated OLS Autoregressive Coefficient

Sample size $T$	Probability that $T(\hat{\rho} - 1)$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
<i>Case 1</i>								
25	-11.9	-9.3	-7.3	-5.3	1.01	1.40	1.79	2.28
50	-12.9	-9.9	-7.7	-5.5	0.97	1.35	1.70	2.16
100	-13.3	-10.2	-7.9	-5.6	0.95	1.31	1.65	2.09
250	-13.6	-10.3	-8.0	-5.7	0.93	1.28	1.62	2.04
500	-13.7	-10.4	-8.0	-5.7	0.93	1.28	1.61	2.04
$\infty$	-13.8	-10.5	-8.1	-5.7	0.93	1.28	1.60	2.03
<i>Case 2</i>								
25	-17.2	-14.6	-12.5	-10.2	-0.76	0.01	0.65	1.40
50	-18.9	-15.7	-13.3	-10.7	-0.81	-0.07	0.53	1.22
100	-19.8	-16.3	-13.7	-11.0	-0.83	-0.10	0.47	1.14
250	-20.3	-16.6	-14.0	-11.2	-0.84	-0.12	0.43	1.09
500	-20.5	-16.8	-14.0	-11.2	-0.84	-0.13	0.42	1.06
$\infty$	-20.7	-16.9	-14.1	-11.3	-0.85	-0.13	0.41	1.04
<i>Case 4</i>								
25	-22.5	-19.9	-17.9	-15.6	-3.66	-2.51	-1.53	-0.43
50	-25.7	-22.4	-19.8	-16.8	-3.71	-2.60	-1.66	-0.65
100	-27.4	-23.6	-20.7	-17.5	-3.74	-2.62	-1.73	-0.75
250	-28.4	-24.4	-21.3	-18.0	-3.75	-2.64	-1.78	-0.82
500	-28.9	-24.8	-21.5	-18.1	-3.76	-2.65	-1.78	-0.84
$\infty$	-29.5	-25.1	-21.8	-18.3	-3.77	-2.66	-1.79	-0.87

The probability shown at the head of the column is the area in the left-hand tail.

Source: Wayne A. Fuller, *Introduction to Statistical Time Series*, Wiley, New York, 1976, p. 371.

**TABLE B.6**  
Critical Values for the Phillips-Perron  $Z_\rho$  Test and for the Dickey-Fuller Test  
Based on Estimated OLS  $t$  Statistic

Sample size $T$	Probability that $(\hat{\rho} - 1)/\hat{\sigma}_\rho$ is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
<i>Case 1</i>								
25	-2.66	-2.26	-1.95	-1.60	0.92	1.33	1.70	2.16
50	-2.62	-2.25	-1.95	-1.61	0.91	1.31	1.66	2.08
100	-2.60	-2.24	-1.95	-1.61	0.90	1.29	1.64	2.03
250	-2.58	-2.23	-1.95	-1.62	0.89	1.29	1.63	2.01
500	-2.58	-2.23	-1.95	-1.62	0.89	1.28	1.62	2.00
$\infty$	-2.58	-2.23	-1.95	-1.62	0.89	1.28	1.62	2.00
<i>Case 2</i>								
25	-3.75	-3.33	-3.00	-2.63	-0.37	0.00	0.34	0.72
50	-3.58	-3.22	-2.93	-2.60	-0.40	-0.03	0.29	0.66
100	-3.51	-3.17	-2.89	-2.58	-0.42	-0.05	0.26	0.63
250	-3.46	-3.14	-2.88	-2.57	-0.42	-0.06	0.24	0.62
500	-3.44	-3.13	-2.87	-2.57	-0.43	-0.07	0.24	0.61
$\infty$	-3.43	-3.12	-2.86	-2.57	-0.44	-0.07	0.23	0.60
<i>Case 4</i>								
25	-4.38	-3.95	-3.60	-3.24	-1.14	-0.80	-0.50	-0.15
50	-4.15	-3.80	-3.50	-3.18	-1.19	-0.87	-0.58	-0.24
100	-4.04	-3.73	-3.45	-3.15	-1.22	-0.90	-0.62	-0.28
250	-3.99	-3.69	-3.43	-3.13	-1.23	-0.92	-0.64	-0.31
500	-3.98	-3.68	-3.42	-3.13	-1.24	-0.93	-0.65	-0.32
$\infty$	-3.96	-3.66	-3.41	-3.12	-1.25	-0.94	-0.66	-0.33

The probability shown at the head of the column is the area in the left-hand tail.

Source: Wayne A. Fuller, *Introduction to Statistical Time Series*, Wiley, New York, 1976, p. 373.

**TABLE B.7**  
Critical Values for the Dickey-Fuller Test Based on the OLS F Statistic

Sample size <i>T</i>	Probability that <i>F</i> test is greater than entry							
	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01
<b>Case 2</b>								
(F test of $\alpha = 0, \rho = 1$ in regression $y_t = \alpha + \rho y_{t-1} + u_t$ )								
25	0.29	0.38	0.49	0.65	4.12	5.18	6.30	7.88
50	0.29	0.39	0.50	0.66	3.94	4.86	5.80	7.06
100	0.29	0.39	0.50	0.67	3.86	4.71	5.57	6.70
250	0.30	0.39	0.51	0.67	3.81	4.63	5.45	6.52
500	0.30	0.39	0.51	0.67	3.79	4.61	5.41	6.47
$\infty$	0.30	0.40	0.51	0.67	3.78	4.59	5.38	6.43
<b>Case 4</b>								
(F test of $\delta = 0, \rho = 1$ in regression $y_t = \alpha + \delta t + \rho y_{t-1} + u_t$ )								
25	0.74	0.90	1.08	1.33	5.91	7.24	8.65	10.61
50	0.76	0.93	1.11	1.37	5.61	6.73	7.81	9.31
100	0.76	0.94	1.12	1.38	5.47	6.49	7.44	8.73
250	0.76	0.94	1.13	1.39	5.39	6.34	7.25	8.43
500	0.76	0.94	1.13	1.39	5.36	6.30	7.20	8.34
$\infty$	0.77	0.94	1.13	1.39	5.34	6.25	7.16	8.27

The probability shown at the head of the column is the area in the right-hand tail.

Source: David A. Dickey and Wayne A. Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica* 49 (1981), p. 1063.

**TABLE B.8**  
Critical Values for the Phillips  $Z_p$  Statistic When Applied to Residuals from Spurious Cointegrating Regression

Number of right-hand variables in regression, excluding trend or constant ( <i>n</i> - 1)	Sample size ( <i>T</i> )	Probability that $(T - 1)(\hat{\rho} - 1)$ is less than entry						
		0.010	0.025	0.050	0.075	0.100	0.125	0.150
<b>Case 1</b>								
1	500	-22.8	-18.9	-15.6	-13.8	-12.5	-11.6	-10.7
2	500	-29.3	-25.2	-21.5	-19.6	-18.2	-17.0	-16.0
3	500	-36.2	-31.5	-27.9	-25.5	-23.9	-22.6	-21.5
4	500	-42.9	-37.5	-33.5	-30.9	-28.9	-27.4	-26.2
5	500	-48.5	-42.5	-38.1	-35.5	-33.8	-32.3	-30.9
<b>Case 2</b>								
1	500	-28.3	-23.8	-20.5	-18.5	-17.0	-15.9	-14.9
2	500	-34.2	-29.7	-26.1	-23.9	-22.2	-21.0	-19.9
3	500	-41.1	-35.7	-32.1	-29.5	-27.6	-26.2	-25.1
4	500	-47.5	-41.6	-37.2	-34.7	-32.7	-31.2	-29.9
5	500	-52.2	-46.5	-41.9	-39.1	-37.0	-35.5	-34.2
<b>Case 3</b>								
1	500	-28.9	-24.8	-21.5	—	-18.1	—	—
2	500	-35.4	-30.8	-27.1	-24.8	-23.2	-21.8	-20.8
3	500	-40.3	-36.1	-32.2	-29.7	-27.8	-26.5	-25.3
4	500	-47.4	-42.6	-37.7	-35.0	-33.2	-31.7	-30.3
5	500	-53.6	-47.1	-42.5	-39.7	-37.7	-36.0	-34.6

The probability shown at the head of the column is the area in the left-hand tail.

Source: P. C. B. Phillips and S. Ouliaris, "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica* 58 (1990), pp. 189-90. Also Wayne A. Fuller, *Introduction to Statistical Time Series*, Wiley, New York, 1976, p. 371.

**TABLE B.9**  
Critical Values for the Phillips  $Z_t$  Statistic or the Dickey-Fuller  $t$  Statistic When Applied to Residuals from Spurious Cointegrating Regression

Number of right-hand variables in regression, excluding trend or constant ( $n - 1$ )	Sample size ( $T$ )	Probability that $(\hat{\rho} - 1)/\hat{\sigma}_\rho$ is less than entry						
		0.010	0.025	0.050	0.075	0.100	0.125	0.150
		<i>Case 1</i>						
1	500	-3.39	-3.05	-2.76	-2.58	-2.45	-2.35	-2.26
2	500	-3.84	-3.55	-3.27	-3.11	-2.99	-2.88	-2.79
3	500	-4.30	-3.99	-3.74	-3.57	-3.44	-3.35	-3.26
4	500	-4.67	-4.38	-4.13	-3.95	-3.81	-3.71	-3.61
5	500	-4.99	-4.67	-4.40	-4.25	-4.14	-4.04	-3.94
<i>Case 2</i>								
1	500	-3.96	-3.64	-3.37	-3.20	-3.07	-2.96	-2.86
2	500	-4.31	-4.02	-3.77	-3.58	-3.45	-3.35	-3.26
3	500	-4.73	-4.37	-4.11	-3.96	-3.83	-3.73	-3.65
4	500	-5.07	-4.71	-4.45	-4.29	-4.16	-4.05	-3.96
5	500	-5.28	-4.98	-4.71	-4.56	-4.43	-4.33	-4.24
<i>Case 3</i>								
1	500	-3.98	-3.68	-3.42	—	-3.13	—	—
2	500	-4.36	-4.07	-3.80	-3.65	-3.52	-3.42	-3.33
3	500	-4.65	-4.39	-4.16	-3.98	-3.84	-3.74	-3.66
4	500	-5.04	-4.77	-4.49	-4.32	-4.20	-4.08	-4.00
5	500	-5.36	-5.02	-4.74	-4.58	-4.46	-4.36	-4.28

The probability shown at the head of the column is the area in the left-hand tail.

Source: P. C. B. Phillips and S. Ouliaris, "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica* 58 (1990), p. 190. Also Wayne A. Fuller, *Introduction to Statistical Time Series*, Wiley, New York, 1976, p. 373.

**TABLE B.10**  
Critical Values for Johansen's Likelihood Ratio Test of the Null Hypothesis of  $h$  Cointegrating Relations Against the Alternative of No Restrictions

Number of random walks ( $g = n - h$ ) ( $g$ )	Sample size ( $T$ )	Probability that $2(\mathcal{L}_A - \mathcal{L}_0)$ is greater than entry					
		0.500	0.200	0.100	0.050	0.025	0.001
<i>Case 1</i>							
1	400	0.58	1.82	2.86	3.84	4.93	6.51
2	400	5.42	8.45	10.47	12.53	14.43	16.31
3	400	14.30	18.83	21.63	24.31	26.64	29.75
4	400	27.10	33.16	36.58	39.89	42.30	45.58
5	400	43.79	51.13	55.44	59.46	62.91	66.52
<i>Case 2</i>							
1	400	2.415	4.905	6.691	8.083	9.658	11.576
2	400	9.335	13.038	15.583	17.844	19.611	21.962
3	400	20.188	25.445	28.436	31.256	34.062	37.291
4	400	34.873	41.623	45.248	48.419	51.801	55.551
5	400	53.373	61.566	65.956	69.977	73.031	77.911
<i>Case 3</i>							
1	400	0.447	1.699	2.816	3.962	5.332	6.936
2	400	7.638	11.164	13.338	15.197	17.299	19.310
3	400	18.759	23.868	26.791	29.509	32.313	35.397
4	400	33.672	40.250	43.964	47.181	50.424	53.792
5	400	52.588	60.215	65.063	68.905	72.140	76.955

The probability shown at the head of the column is the area in the right-hand tail. The number of random walks under the null hypothesis ( $g$ ) is given by the number of variables described by the vector autoregression ( $n$ ) minus the number of cointegrating relations under the null hypothesis ( $h$ ). In each case the alternative is that  $g = 0$ .

Source: Michael Osterwald-Lenum, "A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics," *Oxford Bulletin of Economics and Statistics* 54 (1992), p. 462; and Søren Johansen and Katarina Juselius, "Maximum Likelihood Estimation and Inference on Cointegration—with Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics* 52 (1990), p. 208.